Ciężar D jest przytrzymywany w pionie a sprężyna jest rozciągnięta o wielkość λ i puszczony bez prędkości początkowej. Punkt B zaczyna wykonywać ruch w lewo

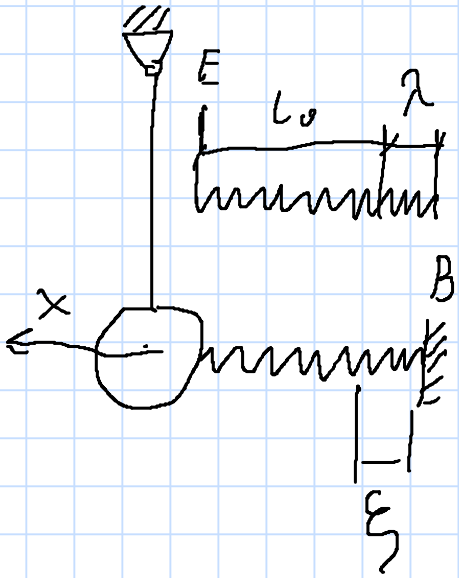
Dane

$m = 3\,kg$

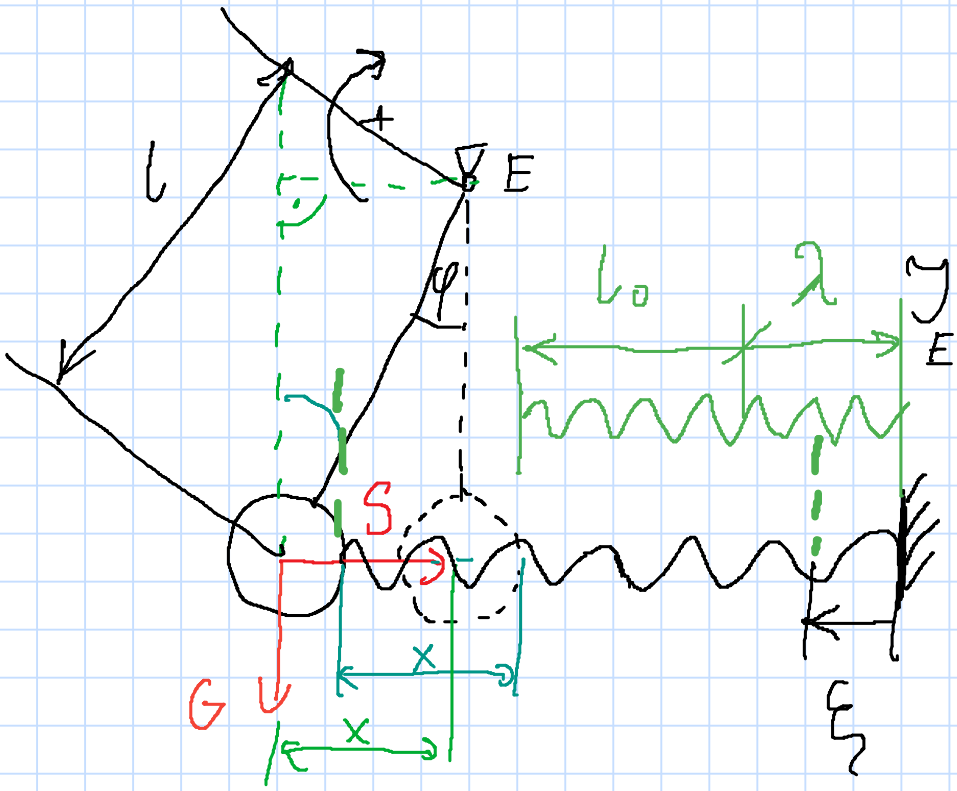
$\lambda = 2\,cm$

$c = 9\,N/cm = 900\,N/m$

$\xi = 1{,}2 \sin(8t)\,[cm]$



Wersja 1



$\sum_E \mathcal{E} = \sum M_E$

$\mathcal{J}_E = m l^2$

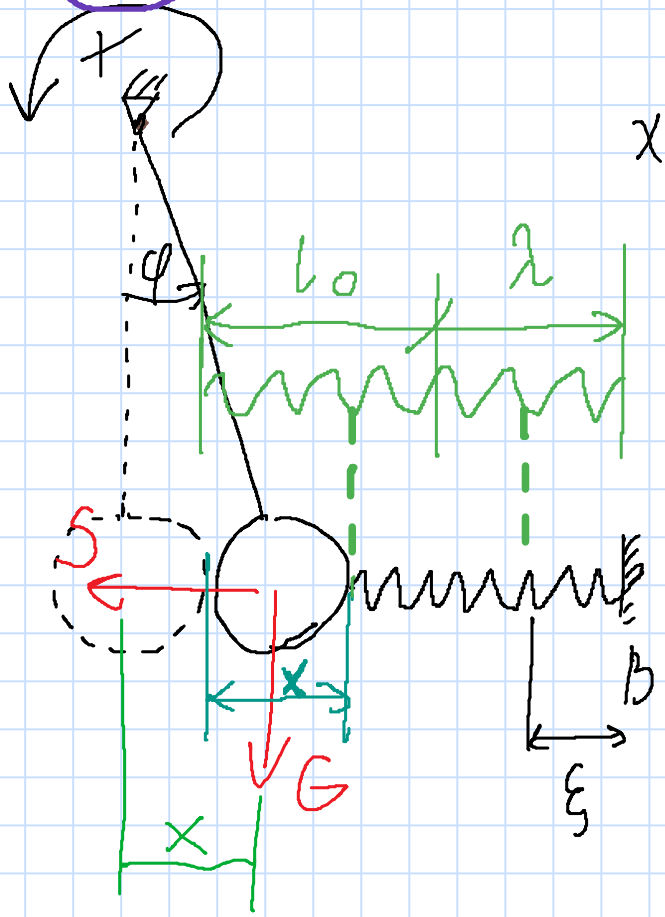
$\sum M_E = -G \cdot l \sin\varphi - S l \cos\varphi$

$G = m \cdot g \qquad S = c \cdot \Delta l$

$\Delta l = x + \lambda - \xi$

$x = l \sin\varphi$

Wersja 2



$$x = l \sin\varphi$$

$$\sum M_E = -G l \sin\varphi - S l \cos\varphi$$

$$S = c\,\Delta l$$

$$\Delta l = x - \lambda + \xi$$

$$m l^2 \ddot{\varphi} = -mgl\sin\varphi - c\left(l\sin\varphi - \lambda + \xi\right)\cdot l\cdot\cos\varphi$$

dla małych kątów

$$\sin\varphi = \varphi$$

$$\cos\varphi = 1$$

$$m l^2 \ddot{\varphi} = -mgl\varphi - c\left(l\cdot\varphi - \lambda + \xi\right)\cdot l$$

$$\ddot{\varphi} + \frac{mgl + cl^2}{ml^2}\cdot\varphi = \frac{-c\lambda l}{ml^2} + \frac{c\,l\,0{,}012\sin 8t}{ml^2}$$

$$\ddot{\varphi} + \frac{mg + cl}{ml}\cdot\varphi = -\frac{c\lambda}{ml} - \frac{c\cdot 0{,}012}{ml}\sin 8t$$

ω^2 $\quad$ k $\quad$ $+$ $\quad$ h $\quad$ p $\quad$ l

$$\varphi = \varphi^{*} + \varphi^{**}$$

częstość drgań układu $\omega = \sqrt{\frac{mg + cl}{ml}}$

$$\ddot{\varphi} + \omega^2\varphi = k + h\sin pt$$

$$\varphi^{*} = A_1\cos\omega t + A_2\sin\omega t$$

$$\begin{cases} \varphi^{**} = D + F\sin L\cdot t \\ \dot{\varphi}^{**} = LF\cos L\cdot t \\ \ddot{\varphi}^{**} = -L^2F\sin L\cdot t \end{cases}$$

$$-L^2F\sin Lt + \omega^2(D + F\sin L\cdot t) = k + h\sin pt$$

$$\omega^2D + (\omega^2F - L^2F)\sin Lt = k + h\sin pt$$

$$L = p$$

$$\omega^2D = k \qquad D = \frac{k}{\omega^2}$$

$$\omega^2F - L^2F = h$$

$$\omega^2F - p^2F = h$$

$$F = \frac{h}{\omega^2 - p^2}$$

$$\varphi^{**} = \frac{k}{\omega^2} + \frac{h}{\omega^2 - p^2}\sin p\cdot t$$

$$\varphi = A_1 \cos \omega t + A_2 \sin \omega t + \frac{k}{\omega^2} + \frac{h}{\omega^2 - p^2} \sin p \cdot t$$

$$\dot{\varphi} = -\omega A_1 \sin \omega t + \omega A_2 \cos \omega t + \frac{h \cdot p}{\omega^2 - p^2} \cos p \cdot t$$

dla $t_0 = 0 \quad \varphi_0 = 0 \quad \dot{\varphi}_0 = 0$

$$0 = A_1 + \frac{k}{\omega^2} + 0 \Rightarrow A_1 = \frac{-k}{\omega^2}$$

$$0 = -\omega A_1 \cdot 0 + \omega A_2 + \frac{h \cdot p}{\omega^2 - p^2}$$

A_2

- wyznaczamy A_2
- podstawiamy do wzorów na φ i $\dot{\varphi}$ stałe A_1 i A_2
- podstawiamy pod k, h, p zależności z poprzedniej strony
- gotowe